

Ex.18 设 $A$  是3 阶方阵且 $|A| = \frac{1}{2}$ , 求行列式 $|(3A)^{-1} - 2A^*|$ .

解法一. 由 $|A| = \frac{1}{2}$  得 $|A^{-1}| = 2$ . 再由 $A^* = |A|A^{-1}$  得

$$\begin{aligned} |(3A)^{-1} - 2A^*| &= \frac{1}{3}|A^{-1}| - 2|A|A^{-1}| \\ &= (\frac{1}{3} - 2|A|)|A^{-1}| \\ &= (\frac{1}{3} - 1) \times 2 \\ &= \left(-\frac{2}{3}\right)^3 \times 2 \\ &= -\frac{16}{27}. \end{aligned}$$

解法二. 由 $AA^* = |A|E$  得

$$\begin{aligned} |(3A)^{-1} - 2A^*| &= \frac{1}{|A|} \cdot |A| \cdot |(3A)^{-1} - 2A^*| \\ &= \frac{1}{|A|} \cdot |\frac{1}{3}AA^{-1} - 2AA^*| \\ &= 2|\frac{1}{3}E - 2|A|E| \\ &= 2|\frac{1}{3}E - E| \\ &= 2|-\frac{2}{3}E| \\ &= 2 \cdot \left(-\frac{2}{3}\right)^3 \\ &= -\frac{16}{27}. \end{aligned}$$